**Discrete Math. Graduation Exam, Spring 2024**

1. (10 points) Prove the following identity.

$$1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + \cdots + (n-1) \cdot 2 + n \cdot 1 = \binom{n+2}{3}.$$

2. Let $m$ and $n$ be positive integers greater than 2. Recall that the *Ramsey Number* $R(m,n)$ is the minimum number of people at a party such that there are either $m$ mutual friends or $n$ mutual enemies, assuming that every pair of people at the party are friends or enemies.

   (a) (5 points) Show that $R(m,n) \leq R(m-1,n) + R(m,n-1)$.
   *Hint.* Let $r = R(m-1,n) + R(m,n-1)$. Consider a party of $r$ people. Choose any person, say A. Let $M$ be the set of friends of A, and $N$ be the set of enemies of A. Then either $M$ has at least $R(m-1,n)$ people or $N$ has at least $R(m,n-1)$ people.

   (b) (5 points) Prove that $R(4,4) \leq 20$. *Hint.* First find an upper bound of $R(4,3)$.

3. (10 points) Are the two graphs below isomorphic? If so, describe the isomorphism. If not, explain why.