

Applied Linear Algebra, 2024

1. (10pts) Show that the equation  $5x^2 + 8xy + 5y^2 = 1$  in  $\mathbf{R}^2$  defines an ellipse and find the major and minor axes.
2. (10pts) Assume that (i) the total population of Pohang and Gyeongju is a constant in times; (ii) for each year 20-percent of Phohang people move to Gyeongju and 30-percent of Gyeongju people move to Phohang. Predict the future distribution of the population.
3. (10pts) Remind that a complex square matrix  $A$  is normal if it commutes with its conjugate transpose  $A^*$ . Show that
  - A normal matrix is unitary if and only if all of its eigenvalues lie on the unit circle of the complex plane.
  - A normal triangular matrix is diagonal.