Calculus Graduation Exam

Spring 2024

1. (10 points) Show that the Taylor series for $\cos x$ at $x = 0$ converges to $\cos x$ for all $x \in \mathbb{R}$.

2. (10 points) Find the points on the surface $z^2 = xy + 1$ that are closest to the origin. (Hint: Use the Lagrange's multiplier method)

3. (10 points) Using Stoke's Theorem, find the circulation of the field $\vec{F}$ around the curve C, where $\vec{F} = (x^2y^3)\vec{i} + \vec{j} + (z)\vec{k}$ and C is the intersection of the cylinder $x^2 + y^2 = 4$ and the hemisphere $x^2 + y^2 + z^2 = 16,\ z \geq 0$, counterclockwise when viewed from above.