

Introduction to Numerical Analysis
Fall, 2023

1. [10 points] Consider Newton's method to find the root of $f(x) = x^2$. The approximation is given by the sequence $\{p_n\}_{n=0}^{\infty}$ with the initial guess $p_0 > 0$. Find p_n in terms of p_{n-1} and determine the order of convergence of p_n .

2. [10 points] Consider the Trapezoidal rule as follows with $h = x_1 - x_0, x_1 > x_0$:

$$\int_{x_0}^{x_1} f(x)dx \approx \frac{h}{2}[f(x_0) + f(x_1)].$$

Show that the absolute error of the rule is $|\frac{h^3}{12}f''(\xi)|, x_0 < \xi < x_1$.

3. [10 points] Consider the well-posed initial-value problem $y' = f(t, y), y(0) = y_0$. Write down the Euler's (explicit) method for the approximation, $w_i \approx y(t_i), i = 1, 2, \dots$ with the uniform step size of h . Suppose that $y'' > 0, t > 0$. Show that w_i underestimates $y(t_i)$ when h is small.