

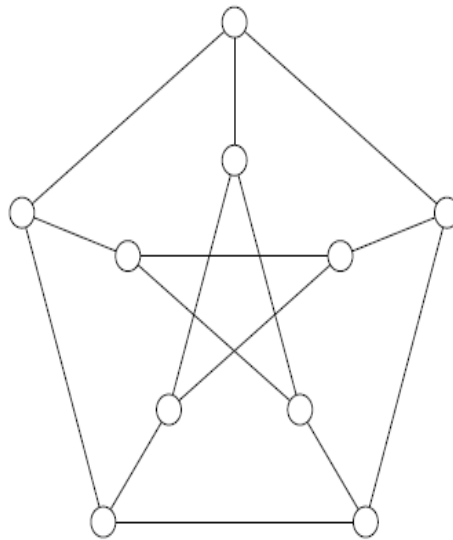
Discrete Math. Graduation Exam, Fall 2023

1. Solve the following recurrence relation.

$$a_0 = 0;$$

$$a_n = \frac{2(n-1)}{n}a_{n-1} + \frac{1}{n}.$$

2. Show that the Petersen graph (figure given below) is not planar. *Hint.* Note that the length of its shortest cycle is five. Or not.



3. Let  $G = (V, E)$  be a simple, undirected weighted graph. Suppose that if edges  $e, f \in E$  are distinct, then the weight of  $e$  and  $f$  are also distinct. Prove that there is a unique minimal spanning tree.