Analysis I - 2023 Graduation Exam

1. (10) Prove that the convergence of $\sum a_n$ implies the convergence of

$$\sum \frac{\sqrt{a_n}}{n},$$

if $a_n \geq 0$.

2. (10) Let X and Y be metric spaces, and let $f : X \to Y$ be a uniformly continuous function. Show that if $\{x_n\}$ is a Cauchy sequence in X, then $f(x_n)$ is a Cauchy sequence in Y.

3. (10) Let $f : [0, 1] \to \mathbb{R}$ be a bounded function. Prove or disprove that f is Riemann integrable if f^2 is Riemann integrable.