Modern Algebra I Graduation exam (Spring, 2023)

Problem 1 (10pts) Let G be a group, H be a subgroup of G and N be a normal subgroup of G. Show the following set

$$HN:=\{hn\mid h\in H,\; n\in N\}$$

is a subgroup of G.

Problem 2 (10pts) Find all the groups of order 8.

Problem 3 (10pts) Determine whether A_4 (the alternating group on 4-letters) contains a normal subgroup or not.