

Discrete Math. Graduation Exam, Spring 2023

1. A semiconductor manufacturer runs  $m$  factories (they all produce the same product). She made a contract with  $n$  electronics companies. Each factory  $i$  is able to manufacture  $s_i$  chips. Every electronics company  $j$  determines a number  $d_j$  of chips that she thinks she will need next month. The manufacturer is able to transport  $c_{i,j}$  chips every month from factory  $i$  to company  $j$ . The semiconductor manufacturer is concerned if the  $m$  factories can handle the demand of the  $n$  electronics companies and asks you to design an algorithm that checks for the upcoming month whether or not there is a way to distribute enough chips from the factories to the companies.  
Design and analyze an algorithm for this problem. You should use a maximum flow algorithm.
2. Let  $G$  be a connected planar simple graph. Show that  $G$  has a vertex of degree not exceeding five. Hint: Try to prove this: every connected, simple planar graph with  $e$  edges and  $v$  vertices, with  $v \geq 3$ , satisfies  $e \leq 3v - 6$ .
3. The **rooted Fibonacci trees**  $T_n$  are defined recursively in the following way.  $T_1$  and  $T_2$  are both the rooted tree consisting of a single vertex, and for  $n = 3, 4, \dots$ , the rooted tree  $T_n$  is constructed from a root with  $T_{n-1}$  as its left subtree and  $T_{n-2}$  as its right subtree.  
If  $n$  is a positive integer, find the number of vertices, leaves, and internal vertices of rooted Fibonacci tree  $T_n$ .