Applied Linear Algebra

1. Note that the numbers $61199, 80389, 23408, 86355, 32623$ are divisible by 19. Is the determinant of the following matrix

$$A = \begin{bmatrix} 6 & 1 & 1 & 9 & 9 \\ 8 & 0 & 3 & 8 & 9 \\ 2 & 3 & 4 & 0 & 8 \\ 8 & 6 & 3 & 5 & 5 \\ 3 & 2 & 6 & 2 & 3 \end{bmatrix}$$

divisible by 19? Justify your answer.

2. Show that if a 5×5 matrix K is skew-symmetric, $K^T = -K$, then K is not invertible.

3. Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

(1) Find all eigenvalues of A.

(2) Find all $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$ such that

$$\lim_{n \to \infty} A^n \mathbf{x} = 0$$