미적분학 **2023-1학기 수학과 졸업시험**

2023 Spring CALCULUS Graduation Exam

1. **(10 points)** Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2} x^{2n}.$$

2. Let $T(x)$ be the Taylor series of $\sin x$ centered at $x = 1$.
   (a) **(5 points)** Write down the first four terms of $T(x)$.
   (b) **(5 points)** Show that $T(x) = \sin x$ for all $x \in \mathbb{R}$.

3. **(10 points)** Let S be a subset of $\mathbb{R}^3$ defined by the equation

$$x^2 + y^3 + z^4 = 5.$$

Find the equation of the tangent plane to S at $(2, 1, 0)$.