CALCULUS GRADUATION TEST/FALL SEMESTER, 2022

Problem 1. Find the local maximum and minimum values of the function f subject to the constraint $g = 0$ (Hint: Use the Lagrange multiplier method). Let $f(x, y) = x^2 + y$ and $g(x, y) = \frac{x^2}{8} + \frac{y^2}{2} - 1 = 0$. Find the local maximum and minimum values of f subject to the constraint $g = 0$.

Problem 2. Write down the definition for the saddle point at a critical point (a, b) of the function f. Discuss the local maximum, minimum and the saddle point for the function $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

Problem 3. Let $\mathbf{F}(x, y) = (-y/(x^2 + y^2), x/(x^2 + y^2))$. Let D be the disk of radius R centered at the origin $(0, 0)$. Evaluate the integral

$$\int_\Gamma \mathbf{F} \cdot \mathcal{T} ds,$$

where $\Gamma = \partial D$ is the boundary of D and oriented in the counterclockwise direction and $\mathcal{T}$ is the unit tangent vector on Γ.