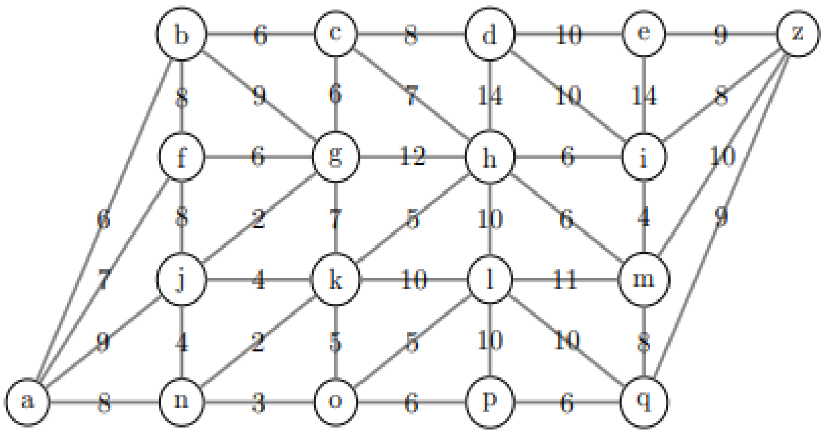
Discrete Math. Graduation Exam, Fall 2022

1. Consider the following weighted graph.



   (a) Find a length of the shortest path from a to z. You do not need to justify your answer.

   (b) The Dijkstra's algorithm is described by the following psuedocode. Given a connected, undirected, weighted graph $G = (V, E)$; and starting vertex a and destination vertex z, it returns a length of a shortest path $L(z)$.

```
function dijkstra(G, a, z)
    L(a) = 0
    for every vertex x other than a do
        L(x) = ∞
    end for
    T = set of all vertices
    while z ∈ T do
        choose v ∈ T with minimum L(v)
        T = T\{v}
        for each x ∈ T adjacent to v do
            L(x) = min(L(x), L(v) + weight of (v, x))
        end for
    end while
    return L(z)
end function
```

   List the elements of the set T when the function *dijkstra* ended. You do not need to justify your answer.

2. Show that the knight's tour puzzle[1] on a 3×4 board does not have a solution.

3. Let $x_1, x_2 \in \mathbb{Z}_2$. The exclusive-OR function $x_1 \oplus x_2$ equals 1 if $x_1 \neq x_2$; equals 0 if $x_1 = x_2$. Find the disjunctive normal form and the conjunctive normal form[2] of $(x_1 \oplus x_2) \oplus x_3$, where $x_1, x_2, x_3 \in \mathbb{Z}_2$.

[1] Place the knight piece on any square of the board. Find a sequence of legal moves, so that the knight piece visits every square exactly once and returns to the original square.

[2] The conjunctive normal form is written by $M_1 \wedge \cdots \wedge M_n$, where $M_i = x_1 \vee \cdots \vee x_m$. The disjunctive normal form is the "opposite" of it.