

Math 230 Probability and Statistics Graduation Exam (Fall 2022)

1. Let  $N(t)$  be the number of customers who have arrived at a grocery store by time  $t$  minutes. We assume that  $N(t)$  follows a Poisson distribution of rate  $\lambda t$ .
  - (a) If the average arrival time of the first customer is 5 min, show that  $\lambda = 0.2$ .
  - (b) What is the probability that only 2 customers have arrived at the grocery store by time  $t = 10$ ?
  
2. Let  $Z$  be the standard normal random variable.
  - (a) Compute  $E(Z^3)$ .
  - (b) Recall that  $g(t) = E(e^{Zt})$  is the moment generating function of  $Z$ . Find  $g'(0)$  and  $g''(0)$ .
  
3. 16 potato chip snack bags are randomly selected from a grocery store. The sample mean weight is 5 grams, and the sample standard deviation is 0.5 grams. Let  $\mu$  denote the average weight of the all potato chip snack bags in the grocery store. Assume that the weight of the potato chip snack bag in the grocery store follows a normal distribution. Then test the null hypothesis  $\mu = 5.5$  with the alternative hypothesis  $\mu < 5.5$  at the 0.01 level of significance. You can use the following table.

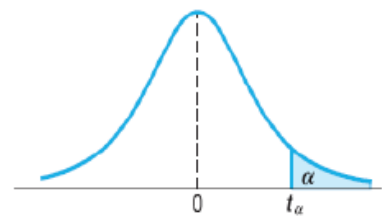


Table A.4 (continued) Critical Values of the  $t$ -Distribution

$v$	$\alpha$						
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015