

Graduate Examination Solution 2022 Spring
Calculus

1. (10pts) Determine convergence of the following series,

(a) (5pts)

$$\sum_{n=1}^{\infty} (-1)^n (\sqrt{n + \sqrt{n}} - \sqrt{n})$$

(b) (5pts)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n} + \sqrt{n+1}}$$

2. (10pts) Find minimum and maximum of $f(x, y, z) = xy + z^2$ on the circle which is given by intersection of the sphere $x^2 + y^2 + z^2 = 4$ and the plane $y = x$.

3. (10 pts) Let S be a closed smooth surface in \mathbb{R}^3 . For smooth vector field $F(x)$, we prove

$$\int_S (\nabla \times F) \cdot \hat{n} dA = 0 \tag{1}$$

(a) (5pts) Prove (1) using divergence theorem.

(c) (5pts) Prove (1) using Stoke's theorem.