Numerical Analysis - May, 2022

1. Consider the following data set of $(x, f(x))$

$$(x_0, f_0) = (1, 2), \quad (x_1, f_1) = (3, 10), \quad (x_2, f_2) = (2, 5).$$

a) Use the Newton's divided difference method for the interpolation as below

$$f_2(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1).$$

b) Show that the permutation does not alter $f[x_0, x_1, x_2]$. That is, show that

$$f[x_0, x_1, x_2] = f[x_1, x_0, x_2] = f[x_0, x_2, x_1] = \cdots.$$

c) Let $f_L(x)$ be the Lagrange interpolation based on the given data. Show that

$$f_2(x) = f_L(x).$$

2. Consider the following initial value problem for $y = y(x)$

$$y' = f(x, y), \quad a \le x \le b, \quad y(a) = \alpha.$$

a) Consider the uniform grid $a = x_0 < x_1 < \cdots < x_N = b$. Let w_i be the numerical solution at x_i. Construct the Euler's method to solve the given initial value problem.
b) Show that the local truncation error is $O(h)$ where h is the grid spacing. Is the global error, $|y(b) - w_N|$ also, $|y(b) - w_N| \sim O(h)$? Why or why not?

3. Suppose that (x_0, f_0) and (x_1, f_1), $x_0 < x_1$ are given and $|f''(x)|$ exists in $[x_0, x_1]$. Use the Lagrange interpolation to find α and β for the integral

$$\int_{x_0}^{x_1} f(x)dx \approx \alpha f_0 + \beta f_1 = A$$

Then show that $\left|\int_{x_0}^{x_1} f(x)dx - A\right| = C \cdot h^3$ where $C < \infty$ is a constant and $h = x_1 - x_0$.