

Graduate Examination 2021 Fall
Analysis

1. Compute

$$\lim_{n \rightarrow \infty} (a_1^n + a_2^n + \cdots + a_m^n)^{\frac{1}{n}}$$

when $a_1 \geq a_2 \geq \cdots \geq a_m \geq 0$.

2. Let A be a compact set in \mathbb{R}^n and B be an open set which contains A , i.e., $A \subset B$.

(a) Show that there exists $\varepsilon > 0$ such that an open ball $B_\varepsilon(x) \subset B$ for any $x \in A$.

(b) Does the statement (a) hold even A is just closed? Prove or provide counter example.

3. Show that the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^2 + n}{n^2}$$

converges uniformly in every bounded intervals, but does not converge absolutely for any value of x .