

Graduation Exam, Applied Complex Variables

Fall, 2021

(1) True or False. Briefly justify your answers.

- (a) The imaginary part of an analytic function in a domain D is a harmonic function on D .
- (b) There is a bounded entire function $f(z)$ with $f'(0) = i$.
- (c) There is an entire function $f(z)$ such that $|f(1)| > |f(z)|$ for all z with $|z| = 2$.
- (d) There is an entire function whose real part is e^x .

(2) Evaluate

$$\int_0^{\infty} \frac{\sin x}{x} dx.$$

(3) Determine the number of roots, counting multiplicities, of the equation

$$z^4 + 3z^3 + 6 = 0$$

inside the circle $|z| = 2$.