수치해석개론 2021-2학기 수학과 졸업시험

Numerical Analysis: Fall, 2021

1. Suppose that $N+1$ distinct points, x_i and the corresponding function values of $f(x_i)$ on x_i, $i = 0, 1, 2, \cdots, N$ are given. Let $f_N(x)$ be the interpolation of $f(x)$ on the given x_i as a polynomial of degree at most N.
a) Let $P_N(x) = \Pi_{i=0}^{N}(x - x_i)$. Using the interpolation condition $f(x_i) = f_N(x_i)$ find the Lagrange polynomials $\ell_i(x)$ in terms of $P_N(x)$ and $P_N'(x)$ where $P_N'(x)$ is the derivative of $P_N(x)$.
b) Show that $f_N(x)$ is unique.

2) Suppose that $f \in C[a, b]$, $f(a) \cdot f(b) < 0$, and $f(p) = 0$, $a < p < b$.
a) Give the sequence generated by the Bisection method to find p.
b) Show that the sequence always converges.

3) Assume that $f(x_i) = \sin(x_i)$, on the uniform grid, $0 = x_0 < x_1 < \cdots < x_i < \cdots < x_N = 2\pi$. Let D be the forward difference matrix for the approximation of $f'(x)$, i.e. $\vec{f'} \approx D\vec{f}$, $\vec{f} = (f(x_0), \cdots, f(x_N))^T$, and $\vec{f'} = (f'(x_0), \cdots, f'(x_N))^T$.
a) Write down the matrix D.
b) Show that D has the eigenvalue $\lambda = 0$ and find the corresponding eigenvector.