

Probability and Statistics

November 2020

1. Suppose that X has probability density function

$$f_X(x) = \begin{cases} x^2, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Compute the cumulative distribution function $F_X(x)$, and use it to evaluate $P(0 < X < 1)$.

2. Let X_1 and X_2 be independent Poisson random variables with densities

$$f_j(x) = \frac{e^{-\lambda_j} \lambda_j^x}{x!}, \quad x = 0, 1, 2, \dots,$$

for $j = 1, 2$, respectively. What is the distribution of $Y = X_1 + X_2$?

3. Suppose that Y has the following mgf.

$$M_Y(t) = \frac{e^t}{4 - 3e^t}, \quad t < -\ln(0.75)$$

- (a) Find $E(Y)$
(b) Find $E(Y^2)$