Applied Complex Analysis
Show all your work!

1. Let $f(z) = \exp(1/z)$. For each positive real number r, describe the set A_r, which is the image of the set $0 < |z| < r$ under the map $f(z)$.

2. Let m be a positive integer. Suppose that $f(z)$ be an entire function such that $|f(z)| \leq |z|^m$ for all $z \in \mathbb{C}$. Show that $f(z) = az^m$ for some $a \in \mathbb{C}$.

3. Evaluate

$$\int_0^{2\pi} \frac{1}{2 + \sin x}\, dx$$