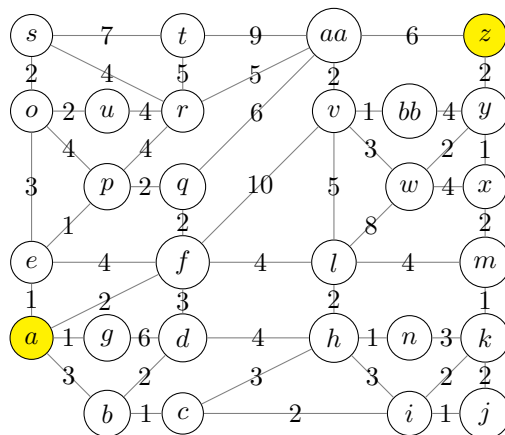


**Discrete Math. Graduation Exam, Fall 2020**

1. Explain Dijkstra's algorithm. Then find the shortest path from the vertex  $a$  to the vertex  $z$ .



2. Prove that if a simple graph with  $n$  vertices has at least

$$\frac{(n-1)(n-2)}{2} + 1$$

edges, then it must be connected.

3. *Dirac's Theorem* states that "if  $G$  is a simple graph with  $n$  vertices with  $n \geq 3$  such that degree of every vertex in  $G$  is at least  $n/2$ , then  $G$  has a Hamilton cycle."

Can you find a simple connected graph with  $n$  vertices with  $n \geq 5$  that does not have a Hamilton cycle, yet the degree of every vertex in the graph is at least  $(n-1)/2$ ?