Graduation Exam-Fall 2020

CALCULUS

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1. Find the radius of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{n^n x^n}{n!} \quad .$$

2. Sketch the polar curve

$$r = 1 - \cos\theta$$

on the xy-plane. And, fine the area of the region enclosed by this curve.

3. Let $\mathcal{S}$ be given by

$$\mathcal{S} : x^2 + y^2 + z^2 = 9, \quad z > 0.$$

And, let $\mathbf{n}$ be the unit normal vector field on $\mathcal{S}$ oriented so that $\mathbf{n} \cdot \hat{k} > 0$ holds on $\mathcal{S}$. Evaluate the surface integral

$$\iint_{\mathcal{S}} \nabla \times \mathbb{F} \cdot \mathbf{n} \, d\sigma$$

for the vector field $\mathbb{F} = y\hat{i} - x\hat{j}$. (Here, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are standard unit vectors in $\mathbb{R}^3$, that is, $\hat{i} = (1, 0, 0)$, $\hat{j} = (0, 1, 0)$, $\hat{k} = (0, 0, 1)$.)