

Math 230 Probability and Statistics Graduation Exam

Spring 2020

1. (10) There are 3 coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

2. (5+5=10) Suppose the joint density function of X and Y is given by

$$f(x, y) = \begin{cases} 24xy, & 0 \leq x \leq 1, 0 \leq y \leq 1, x + y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find $P\{X \leq Y\}$.

(b) Find EX .

3. (10) A study was conducted in which two types of engines, A and B , were compared. Gas mileage, in miles per gallon, was measured. Fifty experiments were conducted using engine type A and 75 experiments were done with engine type B . The gasoline used and other conditions were held constant. The average gas mileage was 36 miles per gallon for engine A and 42 miles per gallon for engine B . Find a 95% confidence interval on $\mu_B - \mu_A$ where μ_A and μ_B are population mean gas mileages for engines A and B , respectively. Assume that the population standard deviations are 6 and 8 for engines A and B respectively. (For the standard normal random variable Z , the point z_α satisfying $\mathbf{P}(Z \geq z_\alpha) = \alpha$ are (i) 1.645 for $\alpha = 0.05$, (ii) 1.96 for $\alpha = 0.025$ and (iii) 2.24 for $\alpha = 0.0125$.)