

**MATH 351 Introduction to Numerical Analysis**

Graduation Exam, 2020 Spring

1. Given the data do the following.

$x$	0	1	2	3
$f(x)$	-4	3	2	5

- Determine the Lagrange interpolating polynomial.
- Construct a divided difference table. Then determine the Newton divided difference interpolating polynomial.
- Find an approximation to  $f(\frac{1}{2})$ .

2. Derive the approximation to  $f'(x)$  based on three points  $x, x + h, x + 2h$  for some  $h > 0$  and show that the error term is  $O(h^2)$  as  $h \rightarrow 0$ .

3. Suppose that the quadrature rule

$$I_4(f) = w_1 f(-1) + w_2 f(-\frac{1}{3}) + w_3 f(\frac{1}{3}) + w_4 f(1)$$

approximates the integral

$$I(f) = \int_{-1}^1 f(x) dx.$$

What choice of the weights  $w_0, w_1, w_2$  and  $w_3$  maximize the degree of precision (accuracy)? What is the degree of precision?