

CALCULUS

1. Evaluate the integral

$$\int_0^{\frac{\sqrt{2}}{2}} x^3 \sqrt{1-x^2} dx.$$

2. Determine the convergence or the divergence of the following series.
You must justify your answer.

$$\sum_{n=10}^{\infty} \frac{1}{n + \sin n}$$

3. For a constant $r > 0$, let \mathcal{H}_r be the sphere in \mathbb{R}^3 of the radius r with the center at the origin $O \in \mathbb{R}^3$. And, let a function $G(r)$ be given by the following surface integral

$$G(r) := \int_{\mathcal{H}_r} \mathbf{F} \cdot \mathbf{n} dA$$

for $\mathbf{F}(x, y, z) = (x, 2y + z^2, -z + x^2 + y^2 + e^{x^2y})$, where \mathbf{n} is the outward unit normal vector field on the sphere \mathcal{H}_r . Evaluate $\frac{dG}{dr}$.