Grduatation Exam, Analysis
2020 Spring, POSTECH

1. (10 pts) Consider a metrix space (X, d) and a Cauchy sequence $\{a_n\}_{n=1}^{\infty}$.
   (a) Show that $\{a_n\}_{n=1}^{\infty}$ is bounded.

   (b) Prove that if a subsequence of $\{a_{n_k}\}$ converges to $a \in X$ then the whole sequence also converges to a either.

2. (10 pts) (a) Consider an infinite subset A of compact set K in a metrix space (X, d). Show that A has a convergent subsequence.

   (b) Use (a) and Problem 1 to show that every Cauchy sequence in $\mathbb{R}^n$ converges.

3. (10 pts) Consider a sequence of Riemann integrable real-valued function f_n on interval $[a, b] \subset \mathbb{R}$. If f_n converges to f uniformly on $[a, b]$, prove the following,

$$\int_a^b f(x)dx = \lim_{n\to\infty} \int_a^b f_n(x)dx.$$