

Analysis

November 2019

1. Let $f_n : \mathbb{R} \rightarrow \mathbb{R}$ be uniformly continuous and let f_n converge uniformly to f . Prove that f is uniformly continuous function.
2. Let $f_n : [0; 1] \rightarrow \mathbb{R}$ be a sequence of equicontinuous functions such that $f_n(x) \rightarrow f(x)$ for every $x \in [0, 1]$. Prove that f_n converges to f uniformly.
3. Let f be defined for all real x , and suppose that

$$|f(x) - f(y)| \leq (x - y)^2$$

for all real x and y . Prove that f is constant.