1. [15] Suppose that two random variables X and Y have the joint probability density

$$f(x,y) = \begin{cases} 2x, & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

   (a) [5] Are X and Y independent?

   (b) [5] Find $P(X > Y)$.

   (c) [5] Calculate $E\left[\dfrac{Y}{X}\right]$.

2. [10] Let X_1 and X_2 be two independent random variables with common mean μ, and variance 1 and 2, respectively. Consider linear combinations $Y_{\boldsymbol{a}} = a_1X_1 + a_2X_2$ for real constants a_1 and a_2.

   (a) [5] Find a condition on $\boldsymbol{a} = (a_1, a_2)$ so that $Y_{\boldsymbol{a}}$ is unbiased for μ.

   (b) [5] Determine $\boldsymbol{a}^* = (a_1^*, a_2^*)$ so that $Y_{\boldsymbol{a}^*}$ has the smallest variance among all unbiased $Y_{\boldsymbol{a}}$.

3. [5] Suppose that a survey showed that only 20 of 100 randomly chosen Postech students said that they are satisfied with the cafeteria meals. Find a 90% confidence interval for the true proportion of satisfied students.

   (For the standard normal random variable Z, the points z_α satisfying $P(Z > z_\alpha) = \alpha$ are (i) 1.28 for $\alpha = 0.10$, (ii) 1.645 for $\alpha = 0.05$, (iii) 1.96 for $\alpha = 0.025$.)