# Introduction to Differential Geometry

Graduation Exam: Fall 2019

Name:

ID Number:

1. Let $\boldsymbol{\alpha}(s)$ be a unit speed regular curve with $\kappa \neq 0$ and Frenet-Serre apparatus $\{\kappa, \tau, \mathbf{T}, \mathbf{N}, \mathbf{B}\}$. Derive the Frenet-Serre equations.

2. Consider the circular helix

$$\boldsymbol{\alpha}(s) = \big(r\cos\omega s, r\sin\omega s, h\omega s\big)$$

where $r$, $h$ and $\omega$ are some constants. When $\boldsymbol{\alpha}(s)$ is a unit speed regular curve and what is the corresponding Frenet-Serre apparatus?

3. Let $M$ be the graph of a smooth function $z = h(x,y)$ in $\mathbb{R}^3$, so that $M$ can be parametized by $\boldsymbol{\sigma}(x,y) = \big(x, y, h(x,y)\big)$. Let $e_1 = \boldsymbol{\sigma}_*\big(\frac{\partial}{\partial x}\big)$ and $e_2 = \boldsymbol{\sigma}_*\big(\frac{\partial}{\partial y}\big)$.
   (a) Find the coefficeints of the the first fundamental form relative to $e_1$, $e_2$.
   (b) Find the coefficeints of the the second fundamental form relative to $e_1$, $e_2$.
   (c) Find the formular for the Gaussian curvature.