

Discrete Math. Graduation Exam, Fall 2019

1. Solve the recurrence relation

$$\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}}$$

with initial conditions $a_0 = a_1 = 1$.

2. A *NAND* gate receives inputs x_1 and x_2 , where x_1 and x_2 are bits, and produces output denoted $x_1 \uparrow x_2$, where

$$x_1 \uparrow x_2 = \begin{cases} 0 & \text{if } x_1 = 1 \text{ and } x_2 = 1 \\ 1 & \text{otherwise} \end{cases}$$

Show that the NAND gate is functionally complete.

3. The *complete bipartite graph* $K_{m,n}$ is a simple graph whose vertices can be divided into two disjoint sets U and V such that

- $|U| = m$ and $|V| = n$
- no two vertices in the same set are adjacent
- each vertex in U is adjacent to all vertices in V ; and each vertex in V is adjacent to all vertices in U .

See Figure 1 for an example.

Choose **at least one** of the following questions and prove it.

- (a) Show that $K_{n,n}$, $n \geq 2$ has a Hamiltonian cycle.
(b) Show that $K_{2,3}$ does not have a Hamiltonian cycle.

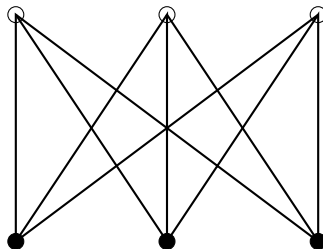


Figure 1: The bipartite graph of $K_{3,3}$. The black dots (resp., white dots) refer the vertices in U (resp., V).