

Graduation examination, Applied Complex Variables

Fall 2019, Postech

1. (10 points) Let  $f(z) = u(x, y) + iv(x, y)$  ( $z = x + iy$ ) be an analytic function in the disk  $D = \{z \in \mathbb{C} : |z| < 2\}$ . Show that the real part  $u(x, y)$  is a harmonic function in the disk  $D_0 = \{(x, y) : x^2 + y^2 < 4\}$  in the plane  $\mathbb{R}^2$ . (First, briefly explain why  $u$  must have all partial derivatives of orders 1 and 2, which are continuous.)

2. (10 points) Evaluate the contour integral

$$\int_C \frac{z^3}{(z+2)^2(z^2+1)} dz$$

where  $C$  is the circle  $|z| = 3$ , described once counterclockwise.

3. (10 points) Use the residue theorem to evaluate the improper integral

$$\int_0^\infty \frac{x^2 + 2}{x^4 + 9} dx.$$