Math 230 Probability and Statistics Graduation Exam

May 2019

1. (10) There are 3 coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

2. (12) Let X be a Poisson random variable with mean $\lambda > 0$, i.e., its probability mass function is given by $f(x) = e^{-\lambda}\frac{\lambda^x}{x!}$ for $x = 0, 1, 2, \ldots$.

   (a) Find the moment generating function $M(t)$ of X.

   (b) Let X and Y be independent Poisson random variables with $\mathrm{E}X = \lambda$ and $\mathrm{E}Y = \mu$. Using (a), show that $X + Y$ is a Poisson random variable with mean $\lambda + \mu$.

3. (8) In a psychological testing experiment, 25 subjects are selected randomly and their reaction time, in seconds, to a particular stimulus is measured. Past experience suggests that the variance in reaction times to these types of stimuli is 4 sec^2 and that the distribution of reaction times is approximately normal. The average time for the subjects is 6 seconds. Give a 95% confidence interval for the mean reaction time.. (For the standard normal random variable Z, the point z_α satisfying $\mathbf{P}(Z \geq z_\alpha) = \alpha$ are (i) 1.645 for $\alpha = 0.05$, (ii) 1.96 for $\alpha = 0.025$ and (iii) 2.24 for $\alpha = 0.0125$.)