

1. Prove that a group in which every element except the identity has order 2 is abelian.
  2. Let  $H = \{\pm 1, \pm i\}$  be the subgroup of  $G = \mathbb{C}^\times$  of fourth roots of unity. Prove that  $G/H$  is isomorphic to  $G$ . Hint: You may want to construct a surjective homomorphism from  $G$  to  $G$  whose kernel is  $H$ . You must show that your map is surjective and a homomorphism.
  3. Let  $G$  and  $G'$  be finite groups whose orders have no common factor. Prove that the only homomorphism  $\varphi : G \rightarrow G'$  is the trivial one  $\varphi(x) = 1$  for all  $x$ .
-