

## Complex variables

May 2019

1. Suppose that  $f(z)$  with  $z \in \mathbb{C}$  is entire and  $u(z) = \operatorname{Re} f(z)$  has an upper bound. Then show that  $u$  is a constant in  $\mathbb{C}$ .

2. Find the number of zeros inside the contour  $C_1 : |z| = 1$ , for

$$h(z) = z^7 - 4z^3 + z - 1.$$

3. Use residues to compute the following integral

$$\int_{-\infty}^{\infty} \frac{\sin x}{x} dx.$$