MATH 311 Analysis I Graduation Exam

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1. Show that $f(x) = \sqrt{x}$ is uniformly continuous on the interval $[0, 1]$.

2. Let f be a continuous real function on a metric space X. Let $Z(f)$ be the set of all $p \in X$ at which $f(p) = 0$ holds. Prove that $Z(f)$ is closed.

3. Let a sequence $\{s_n\}$ be given by

$$s_1 = \sqrt{6}, \quad s_{n+1} = \sqrt{6 + s_n} \qquad (n = 1, 2, 3, \cdots).$$

(a) Prove that the sequence $\{s_n\}$ is bounded above.

(b) Find the limit $\lim_{n\to\infty} s_n$.