Math 230 Probability and Statistics Graduation Exam

Fall 2018

1. (10) Let X be a random variable whose probability density function is given by $f(x) = e^{-2|x|}$ for $x \in (-\infty, \infty)$. Find the cumulative distribution function of X.

2. (10) Let X and Y have joint density $f(x, y) = e^{-x}$ for $0 < y < x$ and 0 otherwise.
   (a) Find $\mathrm{P}(X + Y \leq 2)$.
   (b) Find the conditional probability density function of Y given $X = x > 0$.

3. (10) An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a sample of 25 bulbs has an average life of 780 hours, find a 95% confidence interval for the population mean of all bulbs produced by this firm. (For the standard normal random variable Z, the point z_α satisfying $\mathbf{P}(Z \geq z_\alpha) = \alpha$ are (i) 1.645 for $\alpha = 0.05$, (ii) 1.96 for $\alpha = 0.025$ and (iii) 2.24 for $\alpha = 0.0125$.)