

## MODERN ALGEBRA I

**Problem 1** (10pts). Let  $G$  be a group,  $N$  be a normal subgroup of  $G$  and  $H$  be a subgroup of  $G$ . Show that  $NH$  is a subgroup of  $G$ .

**Problem 2.** Show that every group of order 30 contains a normal subgroup.

**Problem 3.** Prove Cauchy's theorem. That is, for any group  $G$  and prime  $p$  such that  $p \mid |G|$ , show that there exists a subgroup  $H(< G)$  of order  $p$ .