

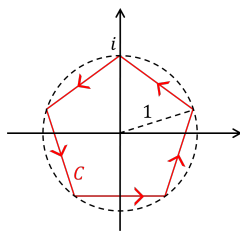
**Total: 30 points**

1. (10 points) Let  $f(z) = u(x, y) + iv(x, y)$  be an analytic function in  $\mathbb{C}$ . Prove that  $u_{yxy}$  is continuous. (Of course, you should not merely state that  $u$  has continuous partial derivatives of all orders.)

2. (10 points) Evaluate the integral

$$\oint_C \frac{\sin 2z}{(6z + \pi)^3} dz,$$

where  $C$  is the pentagonal curve shown below:



3. (10 points) Use the residue theorem to evaluate the improper integral

$$\int_{-\infty}^{\infty} \frac{1}{x^2 - 2 + 2i\sqrt{3}} dx,$$

and express it in the standard form  $a + ib$ .