

## 2018 Complex Analysis QE

Demonstrate enough reasoning and details to establish the following list of facts. You may assume the results of problem A to solve problem B.

1. (10 points) The function  $f(z) = \text{Arg } z$  is continuous at each point  $z = z_0$  of the open plane which does not lie on the negative real half - axis.

2. (10 points)

$$\int_{-\infty}^{\infty} \frac{e^{x/2}}{\cosh x} dx = \sqrt{2}\pi$$

3. (10 points)

$$\pi \cot \pi z = \frac{1}{z} + 2z \sum_{n=1}^{\infty} \frac{1}{z^2 - n^2}$$

4. (10 points)

$$\sin z = z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2\pi^2}\right)$$

5. (15 points)

$$\text{Log}(1 - z) = -\sum_{n=1}^{\infty} \frac{z^n}{n} \quad |z| \leq 1, \quad z \neq 1$$

6. (10 points)

$$\sum_{n=1}^{\infty} \frac{\cos n\theta}{n} = -\text{Log}\left(2 \sin \frac{\theta}{2}\right), \quad 0 < \theta < 2\pi$$

7. (10 points)

$$\sum_{n=1}^{\infty} \frac{\sin n\theta}{n} = \frac{\pi - \theta}{2}, \quad 0 < \theta < 2\pi$$

8. (15 points)

$$\int_0^{2\pi} \text{Log}|1 - ae^{i\theta}| d\theta = 0, \quad 0 \leq a \leq 1$$

9. (10 points)

$$\int_0^{2\pi} \text{Log}|1 - ae^{i\theta}| d\theta = 2\pi \text{Log}|a|, \quad a > 1$$