

MODERN ALGEBRA I (GRADUATING)

Problem 1 (10pts). Let G be a group, H be a subgroup of it, and consider the following set:

$$\{gH \mid g \in G\}.$$

Show that if H is a normal subgroup of G , then the following operation is well-defined on the above set:

$$g_1H \circ g_2H = g_1g_2H.$$

Problem 2 (10pts). (1) Write down the fundamental theorem of group homomorphism.
(2) Prove it.

Problem 3 (10pts). (1) Is S_4 solvable? Prove or disprove it.
(2) Is S_5 solvable? Prove or disprove it.