MATH 351, Introduction to Numerical Analysis **May, 2018**

1. (1) In order to compute numerically a zero of function $f(x)$, explain the main idea of Newton's method and derive the formula of sequence x_{n+1} with unknown x_n.

(2) Compute the first three terms x_1, x_2, and x_3, with the initial term $x_0 = 1$.

(3) Discuss about the speed of the convergence $x_n \to \sqrt{5}$, assuming that x_0 is a good initial guess.

2. Let $p_n(x)$ be the polynomial which interpolates $f(x) = \sin 2x + 3$ at each x_k where $0 = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = \pi$. Prove that

$$\lim_{n\to\infty} p_n(x) = f(x) \qquad \text{for } x \in [0, \pi].$$

3. Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 & 0 \\ 2 & 5 & 5 & 6 & 0 \\ 3 & 5 & 7 & 6 & 0 \\ 0 & 6 & 6 & 8 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{bmatrix}.$$

(1) Find the lower triangular matrix L and the upper triangular matrix U such that $A = LU$ where L has 1's on the main diagonal.

(2) Using the result of (1), find the upper triangular matrix matrix B such that $A = B^T B$.

(3) Using the result of (2), prove that A is a positive definite matrix.

The End