

Introduction to Differential Geometry

May 2018

1. (10pts) Take the unit circle in the xy -plane centered at the origin, and rotate it by $\pi/2$ around the line $y = x$. Express the resulting curve in \mathbb{R}^3 as a parametrized curve $\alpha(t) = (x(t), y(t), z(t))$.
2. (10pts) Define the torsion of a parametrized curve $\alpha(t)$ in \mathbb{R}^3 . Show that the torsion of a plane curve with non-vanishing curvature is zero.
3. (10pts) Suppose S is a regular surface in \mathbb{R}^3 with constant Gaussian curvature -1 . What can you say about the sum of the interior angles of a triangle on S ? Explain with a proof.