

Calculus - May 2018

1. Evaluate the following integrals.

(1) $\int_0^\pi \int_x^\pi \frac{\sin y}{y} dy dx.$

(2) $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} \sqrt{x^2 + y^2 + z^2} dz dy dx.$

2. (1) Evaluate

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}.$$

(2) Find all positive real numbers s such that $\sum_{n=1}^{\infty} (1 - \cos \frac{1}{n^s})$ converges.

3. (a) Find the curve of intersection of the paraboloid $z = x^2 + y^2 + 1$ and the sphere $x^2 + y^2 + z^2 = 5$.

(b) Find the volume of the region D enclosed by the paraboloid $z = x^2 + y^2 + 1$ and the sphere $x^2 + y^2 + z^2 = 5$.