1. (10pts) There is a one-to-one correspondence between a vector (x, y) in xy-plane $\mathbb{R}^2$ and $z = x + iy$ in complex plane $\mathbb{C}$. Moreover we also have $\|(x, y)\|^2 = x^2 + y^2 = |z|^2 = z\overline{z}$. Using the algebraic operations of complex numbers, prove the Pappus's theorem. In other words, for a triangle $\triangle OAB$ with $a = \overline{OA}$, $b = \overline{OB}$ and $2m = \overline{AB}$ and $\ell = \overline{OM}$, where M is the midpoint of A and B, show that

$$a^2 + b^2 = 2(\ell^2 + m^2).$$

2. (7pts for one 10 pts for two) Rewrite the following functions at α using Cauchy integral formula along a simple closed contour C enclosing α.

   (a) $P_n(z) = \dfrac{1}{2^n n!}\dfrac{d^n}{dz^n}(z^2 - 1)^n$

   (b) $L_n(z) = \dfrac{e^z}{n!}\dfrac{d^n}{dz^n}(z^n e^{-z})$

3. (10pts) Find the residue of the function $f(z)$ at $z = -1$:

$$f(z) = \frac{z^2 + 5z + 1}{(z+1)^2(z^2+1)}.$$

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