

Real Analysis

1.1. Prove that the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

converges.

1.2. Prove that if $\sum |a_n|$ converges, then so does

$$\sum_{n=1}^{\infty} \frac{\sqrt{|a_n|}}{n}.$$

2. Prove that for every sequence $\{c_n\}$,

$$\liminf_{n \rightarrow \infty} \frac{|c_{n+1}|}{|c_n|} \leq \liminf_{n \rightarrow \infty} \sqrt[n]{|c_n|}.$$

3. Define

$$f(t) := \int_{-\infty}^{\infty} e^{-x^2} \cos(xt) dx.$$

Prove that f satisfies the differential equation

$$2f'(t) + tf(t) = 0$$

and determine f .