

1. (10) Consider a circle with radius  $X$ , where  $X$  follows the standard uniform distribution with density  $f(x) = 1$ , for  $0 < x < 1$ . Find the distribution function of the area of the circle.

2. (10) Consider a random variable  $X$  with density

$$f(x) = \frac{1}{2}e^{-|x|}, \quad -\infty < x < \infty.$$

(a) (6) Obtain the moment generating function  $m_X(t) = E(e^{tX})$  for  $-1 < t < 1$ .

(b) (4) Find the mean and variance of  $X$ .

3. (10) A survey is to be done to estimate the percentage of citizens in Pohang who favor having their water fluoridated. How large a sample is needed if we wish to be 95% confident that the estimate is within 2% of the true percentage?

(For the standard normal random variable  $Z$ , the points  $z_\alpha$  satisfying  $P(Z < z_\alpha) = \alpha$  is (i) 1.282 for  $\alpha = 0.90$ , (ii) 1.645 for  $\alpha = 0.95$ , and (iii) 1.96 for  $\alpha = 0.975$ .)