

ABSTRACT ALGEBRA 201705(GRADUATION)

Problem

- (1) (10 pt) Let S_n be the n th symmetric group and let A_n be its alternating subgroup. Is A_n a normal subgroup of S_n if $n \geq 2$? Justify your answer.
- (2) (10 pt) Prove the following statement: Let $\phi : G \rightarrow H$ be a group homomorphism for groups G and H .

$$\ker(\phi) = \{e_G\} \text{ iff } \phi \text{ is injective.}$$

Here $\ker(\phi) := \phi^{-1}(\{e_H\})$ and e_G and e_H are identities in G and H , respectively.

- (3) (10 pt) Is $\mathbb{Z}_4 \times \mathbb{Z}_5$ a cyclic group? Justify your answer.

Remark Please note a passing grade ≥ 15 .