

**MATH 351 Introduction to Numerical Analysis**

Graduation Exam  
May, 2017

1. (10 points) We want to find a zero of a function  $f(x)$ . Let  $r$  be a simple zero of  $f$ . Assume that  $f''$  exists and is continuous.

a) Describe Newton's method to find  $r$  and the order of convergence.

b) A student incorrectly recalls Newton's method and writes  $x_{n+1} = f(x_n)/f'(x_n)$ . Will this method find a zero of  $f$ ? If so, what is the order of convergence?

c) Consider an iterative method of the form  $x_{n+1} = x_n - f(x_n)/g(x_n)$ . Assume that it converges to a point  $r$  that is a simple zero of the function  $f$  but not a zero of the function  $g$ . Establish the relationship between  $f$  and  $g$  so that the order of convergence of the method is at least 3.

2. (10 points) Determine appropriate values of  $A_i$  and  $x_i$  so that the quadrature formula

$$\int_{-1}^1 x^2 f(x) dx \approx \sum_{i=0}^n A_i f(x_i)$$

will be correct when  $f$  is any polynomial of degree 3. Use  $n = 1$ .

3. (10 points) Solve the following systems twice. First, use Gaussian elimination and give the factorization  $A = LU$ . Second, use Gaussian elimination with partial pivoting and determine the factorization of the form  $PA = LU$ .

$$\begin{pmatrix} -1 & 1 & -4 \\ 2 & 2 & 0 \\ 3 & 3 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ \frac{1}{2} \end{pmatrix}.$$