

# Introduction to Differential Geometry

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1. Suppose  $\alpha(s)$  is a curve in  $\mathbb{R}^3$  parametrized by arc length  $s$  satisfying  $\alpha''(s) \neq 0$ .
  - (a) Let  $t(s) = \alpha'(s)$  and  $n(s) = t'(s)/|t'(s)|$ . Show that  $t(s)$  and  $n(s)$  are perpendicular unit vectors.
  - (b) Let  $b(s) = t(s) \times n(s)$ . Show that  $b'(s)$  is parallel to  $n(s)$ .
2. Suppose  $\alpha(s)$ ,  $t(s)$ ,  $n(s)$ ,  $b(s)$  are as in #1.
  - (a) Compute  $b(s) \times t(s)$  and  $n(s) \times b(s)$ .
  - (b) Define  $k(s), \tau(s) \in \mathbb{R}$  by  $t'(s) = k(s)n(s)$  and  $b'(s) = \tau(s)n(s)$ . Show that  $n'(s) = -k(s)t(s) - \tau(s)b(s)$ .
3. Compute the Gaussian curvature of the sphere  $x^2 + y^2 + z^2 = r^2$ .