Ph. D. QE—Differentiable Manifolds

June 2017

Do the following problems. Each problem is worth 20 points.

1. Show that the cotangent bundle T^*M of a smooth manifold M of dimension n is itself a smooth manifold of dimension 2n.

2. Find a smooth function $f(x, y)$ such that an ideal inside the Grassmann algebra on $\mathbb{R}^3$ generated by 1-forms $-dx+3y^2dy+dz$ and $ydx+zdy+f(x,y)dz$ can be a differential ideal.

3. Let $f\colon M \to S^7$ is a smooth map from an orientable compact manifold M into the 7-dimensional sphere S^7. Show that for any closed 4-form ω on S^7, it holds that $\int_M f^*\omega = 0$.

4. Explain why S^4 cannot be diffeomorphic to any Lie group.

5. Find the dimension of the Lie group $G = \{A \in GL(n, \mathbb{R})\colon \det A = +1\}$, with a justification.